A Cosmological Exact Solution of Complex Jordan-Brans-Dicke Theory and its Phenomenological Implications

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Abstract

When Brans-Dicke Theory is formulated in terms of the Jordan scalar field φ, dark energy is related to the mass of this field. We investigate the solution which is relevant for the late universe. We show that if φ is taken to be a complex scalar field then an exact solution of the vacuum equations shows that Friedmann equation possesses a term, proportional to the inverse sixth power of the scale factor, as well as a constant term. Possible interpretations and phenomenological implications of this result are discussed.

Keyword(s): Jordan-Brans-Dicke Theory, supernovae Type $1A$, dark energy

1 Introduction

Jordan in 1959 proposed a scalar field φ and replaced the gravitational constant in the Einstein-Hilbert action by a term proportional to φ^2 [1, 2]. Brans and Dicke redefined the scalar field so that $\Box\phi = 0$ as a vacuum solution [3]. Thus, in Jordan-Brans-Dicke theories (JBD), the counterpart of the gravitational coupling term, $1/16\pi G_N$, is replaced by ϕ or $\varphi^2/8\omega$, which may be a function of space and/or time. For an isotropic homogeneous cosmology, which evolves in time, scalar field is a function of only time. JBD gravity has been used extensively to develop dark energy models [4–9] which usually involve a scalar potential which is adjusted to fit observed cosmology. In contrast; we will use a simple model where the JBD field is complex and besides the kinetic term, it contains only a standard mass term for the Jordan field φ. This is in contrast to most cosmological models based on Jordan-Brans-Dicke theory where an arbitrary potential $V(\varphi)$ is assumed and then the potential is adjusted to give the desired solution. One possible motivation for considering a phase for the JBD field is that it could affect the strong QCD parity violating phase χ [10, 11] or the CP violating phase of the quark mass matrix [12, 13] or neutrino mixing [14]. These may be related to dark matter. While in Section II, an exact cosmological solution is displayed and its stability analysis is made for the vacuum case, in section III, it is interpreted as the effect of the proposed complex scalar field to the evolution of the late time universe. We also give a brief discussion of possible relationship of the phase of the complex scalar field. We will show that the dark energy component, in addition to the constant term makes a contribution to the Friedmann equation, which is proportional to the inverse sixth power of the scale factor.

We use a metric signature $(+---)$ and the Jordan formalism. The lagrangian densities of the Jordan and Brans-Dicke language are related by,

$$\begin{aligned} \pounds_{BD} &= -\phi R + \frac{\omega}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \\ &= -\frac{\varphi^2}{8\omega}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = \pounds_{JBD}. \end{aligned} \tag{1}$$

We add a mass term and a phase χ,

$$\varphi = \varphi_1 + i\varphi_2 = |\varphi|\, e^{i\chi} \tag{2}$$

so that the lagrangian density with a complex massive scalar field becomes,

$$\pounds = -\frac{\varphi\varphi^*}{8\omega}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi^* - \frac{1}{2}m^2\varphi\varphi^* \tag{3}$$

which can also be expressed as

$$\begin{aligned}\pounds = -\frac{|\varphi|^2}{8\omega}\left[R - 4\omega g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + 4\omega m^2\right] \\ +\frac{1}{2}g^{\mu\nu}\partial_\mu\left|\varphi\right|\partial_\nu\left|\varphi\right|.\end{aligned} \tag{4}$$

The action is defined by

$$S = \int d^4x\sqrt{-g}\pounds + S_M. \tag{5}$$

When this action is varied with respect to the metric and the complex scalar field, the equations of motion, for a Friedmann-Robertson-Walker metric and perfect fluid energy-momentum tensor $T^\mu_\nu = diag\left(\rho, -p, -p, -p\right)$, reduce to the following:

$$\begin{aligned}\frac{3}{4\omega}\,\varphi\varphi^*\,\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{2}\,\dot{\varphi}\dot{\varphi}^* - \frac{1}{2}\,m^2\,\varphi\varphi^* \\ +\frac{3}{4\omega}\,\frac{\dot{a}}{a}\,(\dot{\varphi}\,\varphi^* + \varphi\dot{\varphi}^*) = \rho_M\end{aligned} \tag{6}$$

$$\begin{aligned}\frac{-1}{4\omega}\varphi\varphi^*\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) - \frac{1}{2\omega}\,\frac{\dot{a}}{a}\,\left(\dot{\varphi}\,\varphi^* + \varphi\dot{\varphi}^*\right) \\ -\frac{1}{4\omega}(\ddot{\varphi}\,\varphi^* + \varphi\ddot{\varphi}^*)\, - \left(\frac{1}{2} + \frac{1}{2\omega}\right)\,\dot{\varphi}\dot{\varphi}^* + \frac{1}{2}\,m^2\,\varphi\varphi^* = p_M\end{aligned} \tag{7}$$

$$\ddot{\varphi} + 3\,\frac{\dot{a}}{a}\,\dot{\varphi} + \left[m^2 - \frac{3}{2\omega}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)\right]\,\varphi = 0. \tag{8}$$

Equation 8, being complex, is equivalent to two real equations. H and F_1+iF_2 are respectively defined as the fractional rate of changes of the scale size and the JBD scalar field φ.

$$H = \frac{\dot{a}}{a} \quad , \quad F_1 + iF_2 = \frac{\dot{\varphi}}{\varphi} \tag{9}$$

where F_1 and F_2 are real, and defined as,

$$F_1 = |\varphi|^{-1} \frac{d|\varphi|}{dt} = \frac{\dot{\varphi}_1\varphi_1 + \dot{\varphi}_2\varphi_2}{\varphi_1^2 + \varphi_2^2} = -\frac{1}{2}\frac{\dot{G}_N}{G_N} \tag{10}$$

$$F_2 = \frac{\dot{\varphi}_2\varphi_1 - \dot{\varphi}_1\varphi_2}{\varphi_1^2 + \varphi_2^2} = \dot{\chi}. \tag{11}$$

For spatially flat (k=0) universe, we will obtain solutions which give H, F_1, F_2 as a function of the scale size a. After rewriting equations 6, 7, and 8 in terms of H, F_1, F_2, and their derivatives with respect to the scale size of the universe, a, we get the following equations:

$$3H^2 - 2\omega(F_1^2 + F_2^2) + 6HF_1 - 2\omega m^2 = \frac{4\omega\rho_M}{|\varphi|^2} \tag{12}$$

$$3H^2 + (2\omega + 4)F_1^2 + 2\omega F_2^2 + 4HF_1 + 2aH(F_1' + H') - 2\omega m^2 = \frac{-4\omega p_M}{|\varphi|^2} \tag{13}$$

$$-6H^2 + 2\omega F_1^2 - 2\omega F_2^2 + 6\omega HF_1 + 2a\omega HF_1' - 3aHH' + 2\omega m^2 = 0 \tag{14}$$

$$(4\omega F_1 + 6\omega H)F_2 + 2\omega aHF_2' = 0 \tag{15}$$

where prime denotes derivative with respect to a.

2 The exact cosmological solutions for dust dominated and the vacuum case

Exact cosmological solutions of JBD theory can be obtained by analyzing symmetries of the field equations [15]. Perturbative solutions were found in [16, 17]. We propose an ansatz for finding the exact solutions for the late time universe. We will show that an exact solution of the cosmological equations can be obtained by an ansatz for the matter dominated case and vacuum case.

$$F_1(a) = \frac{H(a)}{2(\omega + 1)} \tag{16}$$

$$\frac{|\varphi|'}{|\varphi|} = \frac{F_1}{aH} \tag{17}$$

By integrating equation 17 with the ansatz yields

$$|\varphi^2| = |\varphi_0^2| \left[\frac{a}{a_0}\right]^{\frac{1}{(1+\omega)}}. \tag{18}$$

Since $\omega > 10^4$ [18–21] $|\varphi|$ varies very slowly as the universe evolves so φ is approximately constant during matter dominated era. Variation of gravitational constant which is proportional to F_1 in equation 10 is also small due to its proportionality to $\frac{1}{\omega}$.

By using the ansatz, equation 16, equation 15 gives the solution

$$F_2 = F_{20} \left[\frac{a}{a_0}\right]^{\alpha} \tag{19}$$

$$\alpha = -\left(3 + \frac{1}{(1+\omega)}\right) \tag{20}$$

since $\omega > 10^4$ for all practical purposes $\alpha = -3$. For dust solution of complex JBD model, $\rho_M = \rho_0 \left(\frac{a}{a_o}\right)^{-3}$ and $p_M = 0$, when equation 16 and 19 are placed into equation 12, equations 13, 14 are satisfied. Equation 21 is derived as,

$$H^2 = \frac{4\omega(1+\omega)^2}{(3\omega+4)(2\omega+3)} \left[m^2 + \frac{2\rho_0}{|\varphi_0|^2}\left(\frac{a}{a_0}\right)^{\alpha} + F_{20}^2 \left(\frac{a}{a_0}\right)^{2\alpha}\right]. \tag{21}$$

Several interesting features emerge for the vacuum case, $\rho_M = p_M = 0$. Then H^2 contains only a constant term and a $1/a^6$ term, respectively. Since F_2 is directly related to the phase of the scalar field in equation 11, the $1/a^6$ term is identified with the effect of the phase to the expansion of the universe,

$$H^2(a) = \frac{4\omega(1+\omega)^2}{(3\omega+4)(2\omega+3)}(m^2 + F_2^2(a)). \tag{22}$$

When we restrict the model to the $F_2 = 0$ case, $F = F_1$, complex scalar field turns into a real scalar field and previously studied JBD equations are obtained [17]. To test the stability of our vacuum solution; we set

$$a = a(1 + \epsilon\eta) \tag{23}$$

$$\varphi = \varphi(1 + \epsilon\psi). \tag{24}$$

Inserting these variables into Eq.12-15, and after neglecting the higher order terms in ϵ, we obtain four homogeneous differential equations for the three functions η, ψ_R, and ψ_I. We set

$$\eta = \eta_0 e^{\beta t} \tag{25}$$

$$\psi_R = \psi_{R0} e^{\beta t} \tag{26}$$

$$\psi_I = \psi_{I0} e^{\beta t} \tag{27}$$

and obtain four linear homogeneous equations for the three unknowns η_0, ψ_{R0}, and ψ_{I0}. This homogeneous system has a 4x3 matrix of coefficients. The condition that a nontrivial solution exists is that the rank of the matrix of coefficients is at most two. All 3x3 subdeterminants must be zero to obtain a nontrivial solution for β.

We thus obtain four equations for the four 3x3 subdeterminants, and arrange them in powers of β. One equation is cubic in β whereas the other three are quadratic.

$$A_{11}\beta^3 + A_{12}\beta^2 + A_{13}\beta + A_{14} = 0 \tag{28}$$

$$A_{22}\beta^2 + A_{23}\beta + A_{24} = 0 \tag{29}$$

$$A_{32}\beta^2 + A_{33}\beta + A_{34} = 0 \tag{30}$$

$$A_{42}\beta^2 + A_{43}\beta + A_{44} = 0 \tag{31}$$

The determinant of the matrix of coefficients (det A) is nonzero, so we can conclude that there is no solution for β and it means that the solution is stable.

With the addition of the cosmological constant to cold dark matter(CDM) model, the resulting ΛCDM model gives a better fit [22, 23]. Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) temperature and polarization observations [24] which include data from Baryon Acoustic Oscillations in the galaxy and Type Ia supernova luminosity/time dilation measurements [26] are used in the fitting process.

The constant term in equation 21 plays the role of the cosmological constant and we will investigate the extra term from the phenomenological point of view.

3 Phenomenology

For standard cosmology in "matter dominated" era,

$$\frac{H^2}{H_0^2} = \Omega_\Lambda + \Omega_M (\frac{a_0}{a})^3 \tag{32}$$

where Ω_Λ is the fraction of vacuum energy and the matter fraction, Ω_M is interpreted as $\Omega_M = \Omega_{VM} + \Omega_{DM}$ where Ω_{VM} is the fraction of visible matter and Ω_{DM} is the fraction of dark matter. For the complex JBD model, Friedmann equation becomes,

$$\left(\frac{H}{H_0}\right)^2 = \Omega_\Lambda + \Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_\Delta \left(\frac{a_0}{a}\right)^6 . \tag{33}$$

We make the variable transformation and define

$$u = \sqrt{\Omega_\Lambda} \left(\frac{a}{a_0}\right)^3 + \frac{\Omega_M}{2\sqrt{\Omega_\Lambda}} \tag{34}$$

$$c = \sqrt{\mid \Omega_\Delta - \frac{\Omega_M^2}{4\Omega_\Lambda} \mid} \tag{35}$$

$$\varkappa = sgn(\Omega_\Delta - \frac{\Omega_M^2}{4\Omega_\Lambda}) \tag{36}$$

so that Eq.33 is put in differential form

$$3\sqrt{\Omega_\Lambda} H_0 dt = \frac{du}{\sqrt{u^2 + \varkappa c^2)}} . \tag{37}$$

To test the viability of equation 33 predicted by complex JBD model, we have to compare the standard fit to union data [26] with a fit using equation 33 and H_0=71 km/sec/Mpc. With the constraint $\Omega_\Lambda + \Omega_M = 1$, the standard model fit has one free parameter which can be chosen as Ω_Λ.

The latest Type Ia supernova (SNIa) data sets in Table C2 are taken by the Supernova Cosmology Project Group [26]. 414 SNIa supernova magnitude-redshift observations are compiled with "Union" and after selection cuts, it

reduces to 307 Sne. Luminosity distance, in Friedmann-Robertson-Walker Cosmology, is defined in [27]

$$D_L = \frac{c(1+z)}{H_o} \int dz' [\sum \Omega_i (1+z')^{3(1+w_i)} - \kappa_o (1+z')^2]^{\frac{-1}{2}}$$

where $\kappa_o = \sum \Omega_i - 1$.

Distance modulus can be written in terms of luminosity distance

$$\mu = m - M = 5 \log(\frac{D_L}{Mpc}) + 25, \tag{38}$$

where m is the apparent magnitude and M is the absolute magnitude. Three different cases for $\varkappa$ are analyzed. Three fits we would like to present are;

1°) $\varkappa = -1$, Ω_Δ=0 standard cosmology fit with $\Omega_\Lambda + \Omega_M = 1$ gives,

$$\left(\frac{H}{H_0}\right)^2 = 0.745 + 0.255 \left(\frac{a_0}{a}\right)^3 \tag{39}$$

with $\chi^2/d.o.f. = 1.45$. The scale factor and lifetime of universe are related by,

$$\left(\frac{a}{a_0}\right)^3 = \frac{\Omega_M}{2\Omega_\Lambda}(\cosh(3H_0\sqrt{\Omega_\Lambda}t) - 1) \tag{40}$$

2°) $\varkappa = +1$ fit to complex JBD model with $\Omega_M = 0$ and $\Omega_\Lambda + \Omega_\Delta = 1$ gives,

$$\left(\frac{H}{H_0}\right)^2 = 0.938 + 0.062 \left(\frac{a_0}{a}\right)^6 \tag{41}$$

with $\chi^2/d.o.f. = 1.43$. The scale factor and lifetime of universe are related by,

$$\left(\frac{a}{a_0}\right)^3 = \sqrt{\frac{\Omega_\Delta}{\Omega_\Lambda}} \sinh(3H_0\sqrt{\Omega_\Lambda}t). \tag{42}$$

3°) $\varkappa = 0$, fit to complex JBD model with matter, $\Omega_\Lambda + \Omega_M + \Omega_\Delta = 1$ and $\Omega_\Delta = \frac{\Omega_M^2}{4\Omega_\Lambda}$ so that equation 33 becomes

$$\frac{H}{H_0} = \sqrt{\Omega_\Lambda} + \sqrt{\Omega_\Delta}(\frac{a_0}{a})^3 \tag{43}$$

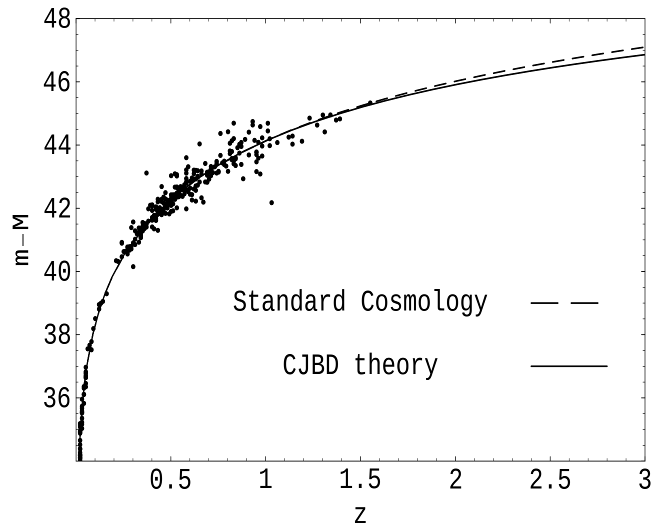

Figure 1: Magnitude vs. Redshift Graph

gives,

$$\left(\frac{H}{H_0}\right)^2 = 0.790 + 0.200\left(\frac{a_0}{a}\right)^3 + 0.010\left(\frac{a_0}{a}\right)^6 \tag{44}$$

with $\chi^2/d.o.f. = 1.43$. This condition makes the r.h.s. of Friedmann equation perfect square and gives the best fit to supernovae union data. The scale factor and lifetime of universe are related by,

$$\left(\frac{a}{a_0}\right)^3 = \frac{\Omega_M}{2\Omega_\Lambda}(\exp(3H_0\sqrt{\Omega_\Lambda}t) - 1) \tag{45}$$

The fit for equation 41 is interesting but unacceptable since the model estimated lifetime is approximately 10 Gyrs, small compared with the observations [25]. In fig.1, standard cosmology fit using Eq.(39) and complex JBD fit Eq.(44) to Union data sets [26] are shown. We conclude that the extra term, from the change of the JBD field's phase,which must be small, indicates a slightly better fit. The difference between two models will be seen for high redshift observations. Similar to the addition of cosmological constant in ΛCDM, cosmological reason of obtaining the phase term in JBD theory can be investigated.

4 Conclusion and Acknowledgements

Complex JBD model which in addition to the constant term makes a contribution of $\frac{1}{a^6}$ term to the Friedmann Equation fits the Supernovae data accurately. It is clear from 22, which is valid for $\rho_M = p_M = 0$, that this term is a natural component of dark energy. Actually we conclude that the complex JBD model explains the evolution of the universe with a slightly better fit. However; the lifetime of the universe was found approximately 12 Gyrs, smaller than the experimental age of the universe, 13,8 Gyrs [25]. Moreover; the complex scalar field is responsible in large scale for the expansion behavior of universe and its phase behaves as a phenomenological extreme density term. The more the density due to complex phase becomes, a smaller model-based age is determined. In standard cosmology, the $\frac{1}{a^6}$ term would be obtained if the kinetic term of a scalar field dominates the energy-momentum tensor. In our model, this is also how it mathematically arises. However, the physical interpretation is that it is, as a typical feature of CJBD theory, a component of dark energy and its presence may be determined by more accurate measurements of the Supernova data near $z \approx 3$. This is far from the radiation dominated age, $z \approx 1100$. This research is in part supported by The Turkish Academy of Sciences, TUBA.

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